Chapter 7 • The Logarithmic Function and Logarithms

Review of Prerequisite Skills

4. The increase in population is given by

$f(x) = 2400(1.06)^x$

$f(20) = 2400(1.06)^{20}$

$\doteq 7697.$

The population in 20 years is about 7700.

5. The function representing the increase in bacteria population is $f(t) = 2000(2^{\frac{t}{4}})$. Determine t when $f(t) = 512\,000$:

$512\,000 = 2000(2^{\frac{t}{4}})$

$256 = 2^{\frac{t}{4}}$

$\frac{t}{4} = 8$

$t = 32.$

The bacteria population will be 512 000 in 32 years.

6. **a.** The function is $A(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}$.

When $t = 200$, $A(t) = 5\left(\frac{1}{2}\right)^{\frac{200}{1620}}$

$= 4.59.$

They will have 4.59 g in 200 years.

b. Determine t when $A(t) = 4$.

$$4 = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{1620}} = 0.8$$

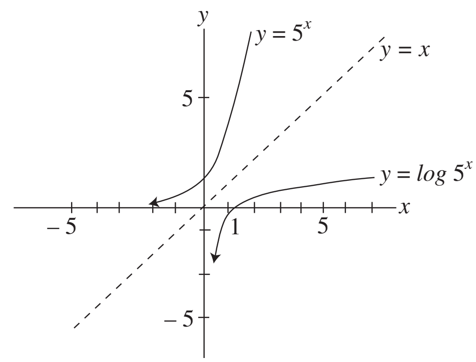
$$\frac{t}{1620}\log 0.5 = \log 0.8$$

$$t = 1620\,\frac{\log 0.8}{\log 0.5}$$

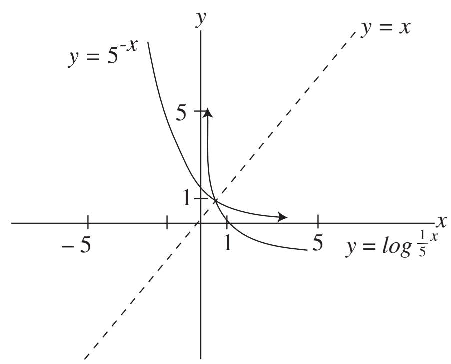
$$= 521.52$$

Exercise 7.1

4.



5.



6. **a.** Let $\log_2 8 = x$.

Then $2^x = 8$, by definition.

But $2^3 = 8$.

$\therefore x = 3$

So, $\log_2 8 = 3$.

c. Let $\log_3 81 = x$.

$3^x = 81$

$\therefore x = 4$

So, $\log_3 81 = 4$.

e. Let $\log_2\left(\frac{1}{8}\right) = x$.

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$x = -3$

So, $\log_2\left(\frac{1}{8}\right) = -3$.

g. Let $\log_5\sqrt{5} = x$.

$$\therefore\ 5^x = \sqrt{5}$$
$$5^x = 5^{\frac{1}{2}}$$
$$x = \frac{1}{2}$$

So, $\log_5\sqrt{5} = \frac{1}{2}$.

i. Let $\log_2\sqrt[4]{32} = x$.

$$\therefore\ 2^x = \sqrt[4]{32}$$
$$2^x = (32)^{\frac{1}{4}}$$
$$2^x = (2^5)^{\frac{1}{4}}$$
$$2^x = 2^{\frac{5}{4}}$$
$$\therefore\ x = \frac{5}{4}$$

So, $\log_2\sqrt[4]{32} = \frac{5}{4}$.

7. **a.** $\log_6 36 - \log_5 25$
$= 2 - 2$
$= 0$

b. $\log_9\left(\frac{1}{3}\right) + \log_3\left(\frac{1}{9}\right)$

$= \log_9(3^{-1}) + \log_3(9^{-2})$

$= \log_9\left[(9)^{\frac{1}{2}}\right]^{-1} + \log_3[(3^2)]^{-2}$

$= \log_9 9^{-\frac{1}{2}} + \log_3(3^{-4})$

$= -\ \frac{1}{2} + (-4)$

$= -4\frac{1}{2}$ or $-\frac{9}{2}$ or -4.5

c. $\log_6\sqrt{36} - \log_{25}5$

$= \log_6(36)^{\frac{1}{2}} - \log_{25}(25^{\frac{1}{2}})$

$= \log_6 6 - \log_{25}(25^{\frac{1}{2}})$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

d. $\log_3\sqrt[4]{27}$

$= \log_3(27)^{\frac{1}{4}}$

$= \log_3(3^3)^{\frac{1}{4}}$

$= \log_3 3^{\frac{3}{4}}$

$= \frac{3}{4}$

e. $\log_3(9 \times \sqrt[5]{9})$

$= \log_3(3^2 \times 9^{\frac{1}{5}})$

$= \log_3(3^2 \times 3^{\frac{2}{6}})$

$= \log_3 3^{\frac{12}{5}}$

$= \frac{12}{5}$ by definition of logarithms

f. $\log_2 16^{\frac{1}{3}}$

$= \log_2(2^4)^{\frac{1}{3}}$

$= \log_2 2^{\frac{4}{3}}$

$= \frac{4}{3}$

8. **b.** $\log_4 x = 2$
$x = 4^2$
$x = 16$

d. $\log_4\left(\frac{1}{64}\right) = x$

$$4^x = \frac{1}{64}$$
$$4^x = 4^{-3}$$
$$x = -3$$

f. $\log_{\frac{1}{4}} x = -2$

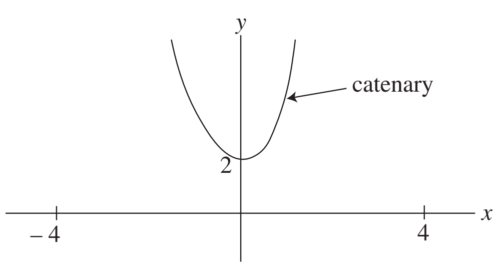
$$x = \left(\frac{1}{4}\right)^{-2}$$
$$= 4^2$$
$$= 16$$

9. To find the value of a logarithm, you can use the $\boxed{\text{LOG}}$ button on a calculator for those which have base 10. The log can be rounded to the number of decimals appropriate to the problem.
If, however, the number can be expressed as a power of the base of the logarithm, then the exponent of the power *is* the logarithm. Since 16 can be written as 2^4, $\log_2 16 = \log_2 2^4$, = 4. By definition $\log_b b^x = x$.
If the number cannot be so expressed, we can use the calculator to find $\log_a b$ by finding

$$\frac{\log_{10} b}{\log_{10} a} \bullet \text{ i.e., } \log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$$
$$= 4 \text{ by calculator.}$$

10.

x	y
± 4	$81\frac{4}{81}$
± 3	$27\frac{1}{27}$
± 2	$9\frac{1}{9}$
± 1	$3\frac{1}{3}$
0	2



11. Integer values for y exist where x is a power of 10 with an integer exponent. If $y > -20$, the smallest number is 10^{-19}. If $x \le 1000$, the largest number is $1000 = 10^3$. There are integers from –19 to 3 that satisfy the condition, so there are $3 - (-19) + 1 = 23$ integer values of y possible.

Section 7.2

Investigation:

2. $\log_b 1 = 0$

3. $\log_b b = 1$

4. $\log_b b^x = x$

5. $b^{\log_b x} = x$

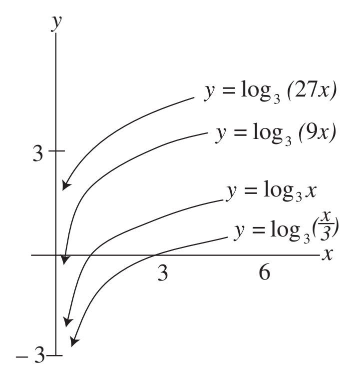
Exercise 7.2

6. **a.** $y = \log_3 x$

b. $y = \log_3(9x)$
$= \log_3 9 + \log_3 x$
$= 2 + \log_3 x$

c. $y = \log_3(27x)$
$= \log_3 27 + \log_3 x$
$= 3 + \log_3 x$

d. $y = \log_3\left(\frac{x}{3}\right)$
$= \log_3 x - 1$



7. **a.** $y = \log_3 x$

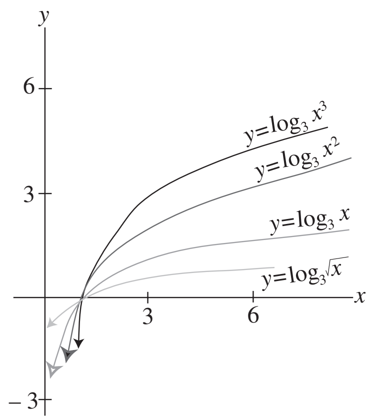
b. $y = \log_3 x^2$
$= 2\log_3 x$

c. $y = \log_3 x^3$
$= 3\log_3 x$

d. $y = \log_3\sqrt{x}$

$= \log_3 x^{\frac{1}{2}}$

$= \frac{1}{2}\log 3$



8. **a.** $\log_3 135 - \log_3 5$

$= \log_3 \frac{135}{5}$

$= \log_3 27$

$= 3$

b. $\log_2 40 + \log_2\left(\frac{4}{5}\right)$

$= \log_2\left(40 \times \frac{4}{5}\right)$

$= \log_2 32$

$= 5$

c. $\log_8 640 - \log_8 10$

$= \log_8 \frac{640}{10}$

$= \log_8 64$

$= 2$

d. $\log_5(2.5) + \log_5 10$

$= \log_5(2.5 \times 10)$

$= \log_5(25)$

$= 2$

e. $\log_2 224 - \log_2 7$

$= \log_2 \frac{224}{7}$

$= \log_2 32$

$= 5$

f. $\log_3 36 + \log_3\left(\frac{3}{4}\right)$

$= \log_3\left(36 \times \frac{3}{4}\right)$

$= \log_3 27$

$= 3$

9. **a.** $\log_3 3 + \log_5 1$

$= 1 + 0$

$= 1$

b. $\log_3 18 + \log_3\left(\frac{3}{2}\right)$

$= \log_3\left(18 \times \frac{3}{2}\right)$

$= \log_3 27$

$= 3$

c. $\log_4 16 - \log_4 1$

$= 2 - 0$ or $\log_4 16 - \log_4 1$

$= 2$ $\log_4(16 \times 1)$

$= 2$

d. $\log_5 5^3$

$= 3$

e. $\log_2 40 - \log_2\left(\frac{5}{2}\right)$

$= \log_2\left(40 \div \frac{5}{2}\right)$

$= \log_2\left(40 \times \frac{2}{5}\right)$

$= \log_2 16$

$= 4$

f. $\log_4 4^4 + \log_3 3^3$

$= 4 + 3$

$= 7$

g. $\log_2 14 + \log_2\left(\frac{4}{7}\right)$

$= \log_2\left(14 \times \frac{4}{7}\right)$

$= \log_2 8$

$= 3$

h. $\log_5 200 - \log_5 8$

$= \log_5\left(\frac{200}{8}\right)$

$= \log_5 25$

$= 2$

10. a. $\log_a \sqrt[3]{x^2y^4}$

$= \log_a (x^2y^4)^{\frac{1}{3}}$

$= \log_a (x^{\frac{2}{3}} \bullet y^{\frac{4}{3}})$

$= \log_a x^{\frac{2}{3}} + \mathit{log}_a y^{\frac{4}{3}}$

$= \frac{2}{3}\log_a x + \frac{4}{3}\log_a y$

b. $\log_a \sqrt{\frac{x^3y^2}{w}}$

$= \log_a (x^3y^2w^{-1})^{\frac{1}{2}}$

$= \frac{1}{2}\log_a (x^3y^2w^{-1})$

$= \frac{1}{2}[\log_a x^3 + \log_a y^2 + \log_a w^{-1}]$

$= \frac{1}{2}[3\log_a x + 2\log_a y - \log_a w]$

c. $\log_a \frac{x^3y^4}{\sqrt{x^{\frac{1}{4}}y^{\frac{2}{3}}}}$

$= \log_a\left[\frac{x^3y^4}{(x^{\frac{1}{4}}y^{\frac{2}{3}})^{\frac{1}{2}}}\right]$

$= \log_a\left(\frac{x^3y^4}{x^{\frac{1}{8}}y^{\frac{1}{3}}}\right)$

$= \log_a\left(x^{\frac{23}{8}}y^{\frac{11}{3}}\right)$

$= \log_a x^{\frac{23}{8}} + \log_a y^{\frac{11}{3}}$

$= \frac{23}{8}\log_a x + \frac{11}{3}\log_a y$

d. $\log_a\left(\frac{x^5}{y^3}\right)^{\frac{1}{4}}$

$= \frac{1}{4}\log_a (x^5y^{-3})$

$= \frac{1}{4}[\log_a x^5 + \log_a y^{-3}]$

$= \frac{1}{4}[5\log_a x - 3\log_a y]$

11. a.
$$\begin{aligned} 10^{2x} &= 495 \\ \log 10^{2x} &= \log 495 \\ 2x \log 10 &= \log 495 \\ 2x &= 495 \\ x &= \log \frac{495}{2} \\ &\doteq 1.347 \end{aligned}$$

b.
$$\begin{aligned} 10^{3x} &= 0.473 \\ \log 10^{3x} &= \log 0.473 \\ 3x\log 10 &= \log 0.473 \\ x &= \frac{\log 0.473}{3} \\ &\doteq -0.1084 \end{aligned}$$

c.
$$\begin{aligned} 10^{-x} &= 31.46 \\ \log 10^{-x} &= 31.46 \\ -x\log 10 &= 31.46 \\ -x &= 31.46 \\ x &= -31.46 \end{aligned}$$

d.
$$\begin{aligned} 7^x &= 35.72 \\ x\log 7 &= 35.72 \\ x &= \frac{\log 35.72}{\log 7} \\ &\doteq 1.8376 \end{aligned}$$

e.
$$\begin{aligned} (0.6)^{4x} &= 0.734 \\ \log(0.6)^{4x} &= \log 0.734 \\ 4x\log 0.6 &= \log 0.734 \\ x &= \frac{\log 0.734}{4\log 0.6} \\ &\doteq 0.1513 \end{aligned}$$

f.
$$
\begin{aligned}
(3.482)^{-x} &= 0.0764\\
\log 3.482^{-x} &= \log 0.0764\\
-x \log 3.482 &= \log 0.0764\\
x &= \log \frac{0.0764}{-\log 3.482}\\
&\doteq 2.0614
\end{aligned}
$$

12. b.
$$
\begin{aligned}
7^{x+9} &= 56\\
\log 7^{x+9} &= \log 56\\
(x+9)\log 7 &= \log 56\\
x+9 &= \frac{\log 56}{\log 7}\\
x &= \frac{\log 56}{\log 7} - 9\\
&\doteq -6.93
\end{aligned}
$$

c.
$$
\begin{aligned}
5^{3x+4} &= 25\\
5^{3x+4} &= 5^2\\
3x+4 &= 2\\
3x &= -2\\
x &= -\frac{2}{3}
\end{aligned}
$$

d.
$$
\begin{aligned}
10^{2x+1} &= 95\\
\log 10^{2x+1} &= \log 95\\
(2x+1)\log 10 &= \log 95\\
2x+1 &= \log 95\\
2x &= \log 95 - 1\\
x &\doteq 0.4889
\end{aligned}
$$

e.
$$
\begin{aligned}
6^{x+5} &= 71.4\\
(x+5)\log 6 &= \log 71.4\\
x+5 &= \frac{\log 71.4}{\log 6}\\
x &\doteq -2.6163
\end{aligned}
$$

f.
$$
\begin{aligned}
3^{5-2x} &= 875\\
(5-2x)\log 3 &= \log 875\\
5-2x &= \frac{\log 875}{\log 3}\\
5-2x &\doteq 6.1662\\
-2x &\doteq 1.662\\
x &\doteq -0.5841
\end{aligned}
$$

13. a.
$$
\begin{aligned}
2 \times 3^x &= 7 \times 5^x\\
\frac{3^x}{5^x} &= \frac{7}{2}\\
\left(\frac{3}{5}\right)^x &= 3.5\\
x\log(0.6) &= \log 3.5\\
x &= \frac{\log 3.5}{\log 0.6}\\
&\doteq -2.452
\end{aligned}
$$

b. $12^x = 4 \times 8^{2x}$

Solution 1

$$
\begin{aligned}
x\log 12 &= \log 4 + 2x\log 8\\
x\log 12 - 2x\log 8 &= \log 4\\
x(\log 12 - 2\log 8) &= \log 4\\
x\left(\log \frac{12}{8^2}\right) &= \log 4\\
x\left(\log \frac{3}{16}\right) &= \log 4\\
x &= \frac{\log 4}{\log \frac{3}{16}} \text{ or } x \doteq -0.828
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
12^x &= 2^2 \times (2^3)^{2x}\\
12^x &= 2^{6x+2}\\
x\log 12 &= (6x+2)\log 2\\
x(\log 12 - 6\log 2) &= 2\log 2\\
x(\log \frac{12}{2^6}) &= 2\log 2\\
x &= \frac{2\log 2}{\log \frac{2}{16}} \text{ or } x = -0.828
\end{aligned}
$$

c. $4.6 \times 1.06^{2x+3} = 5 \times 3^x$

$$
\begin{aligned}
\log 4.6 + (2x+3)\log 1.06 &= \log 5 + x\log 3\\
\log 4.6 + 2x\log 1.06 + 3\log 1.06 &= \log 5 + x\log 3\\
x(2\log 1.06 - \log 3) &= \log 5 - \log 4.6 - 3\log 1.06\\
x\left(\log \frac{1.06^2}{3}\right) &= \log\left(\frac{5}{4.6 \times 1.06^3}\right)\\
x &\doteq \frac{\log(0.9126)}{\log(0.3745)}\\
x &\doteq 0.093
\end{aligned}
$$

d. $2.67 \times 7.38^x = 9.36^{5x-2}$

Solution 1

$$\begin{aligned} 2.67 \times 7.38x &= 9.365\ x - 2 \\ \log 2.67 + x\log 7.38 &= (5x - 2)\log 9.36 \\ \log 2.67 + x\log 7.38 &= 5x\log 9.36 - 2\log 9.36 \\ x(\log 7.38 - 5\log 9.36) &= -\log 2.67 - 2\log 9.36 \\ x\left(\log \frac{7.38}{9.36^5}\right) &= -\log(2.67 \times 9.36^2) \\ x(\log .000103) &\doteq -\log(233.9) \\ x &\doteq 0.59 \end{aligned}$$

Solution 2

$$\begin{aligned} 2.67 \times 7.38^x &= 9.36^{5x-2} \\ 2.67 &= \frac{9.36^{5x-2}}{7.38^x} \\ \log 2.67 &= \log 9.36^{5x-2} - \log 7.38^x \\ \log 2.67 &= (5x - 2)\log 9.36 - x\log 7.38 \\ \log 2.67 &= 5x\log 9.36 - 2\log 9.36 - x\log 7.38^x \\ \log 2.67 + 2\log 9.36 &= x(5\log 9.36 - \log 7.38) \\ \log(2.67 \times 9.36^2) &= x\left(\log \frac{9.36^5}{7.38}\right) \\ \log(233.913) &\doteq x(\log 9734.707) \\ x &\doteq \frac{\log 233.918}{\log 9734.707} \\ x &\doteq 0.59 \end{aligned}$$

e. $12 \times 6^{2x-1} = 11^{x+3}$

Solution 1

$$\begin{aligned} \log 12 + (2x - 1)\log 6 &= (x + 3)\log 11 \\ \log 12 + 2x\log 6 - \log 6 &= x\log 11 + 3\log 11 \\ x(2\log 6 - \log 11) &= 3\log 11 - \log 12 + \log 6 \\ x\left(\log \frac{62}{11}\right) &= \log\left(\frac{11^3 \times 6}{12}\right) \\ x(\log 3.273) &\doteq \log(6665.5) \\ x &\doteq 5.5 \end{aligned}$$

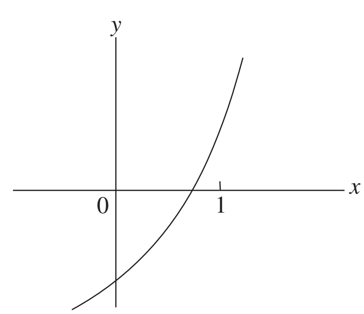
Solution 2

$$\begin{aligned} 12 \times 6^{2x-1} &= 11^{x+3} \\ 12 \times \frac{6^{2x}}{6} &= 11^x \bullet 11^3 \\ 2 \bullet 6^{2x} &= 1331 \bullet 11^x \\ \frac{6^{2x}}{11^x} &= 665.5 \\ 2x\log 6 - x\log 11 &= \log 665.5 \\ x\left(\log \frac{6^2}{11}\right) &= \log 665.5 \\ x &\doteq 5.5 \end{aligned}$$

f.
$$\begin{aligned} 7 \times 0.43^{2x} &= 9 \times 6^{-x} \\ 6^x \times 0.43^{2x} &= \frac{9}{7} \\ x\log 6 + 2x\log 0.43 &\doteq \log 1.2857 \\ x(\log 6 + 2\log 0.43) &\doteq \log 1.2857 \\ x(\log(6 \times 0.43^2)) &\doteq \log 1.2857 \\ x &\doteq 2.42 \end{aligned}$$

g. $5^x + 3^{2x} = 92$
Since we cannot take logarithms of a sum,
let $y = 5^x + 3^{2x} - 92$.
Graphing on a graphing calculator,
when $y = 0$, $x \doteq 1.93$.

h. $4 \times 5^x - 3(0.4)^{2x} = 11$
Let $y = 4 \times 5^x - 3x(0.4)^{2x} - 11$.
Graphing on a graphing calculator to find the value of x for $y = 0$, we find $x \doteq 0.64$.



14. a. $\frac{1}{3}\log_a x + \frac{1}{4}\log_a y - \frac{2}{5}\log_a w$

$= \log_a \sqrt[3]{x} + \log_a \sqrt[4]{y} - \log_a w^{\frac{2}{5}}$

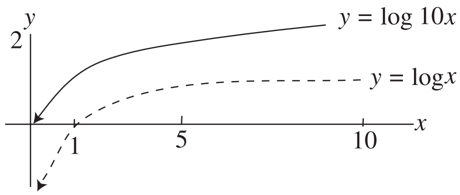
$= \log_a\left[\frac{\sqrt[3]{x}\ \sqrt[4]{y}}{\sqrt[5]{w^2}}\right]$

b. $(4\log_5 x - 2\log_5 y) \div 3\log_5 w$

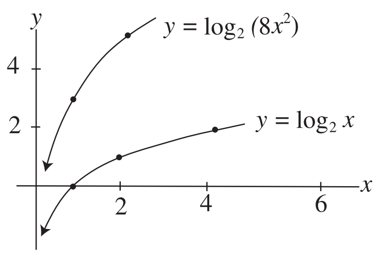
$= (\log_5 x^4 - \log_5 y^2) \div \log_5 w^3$

$= \log_5\left(\dfrac{x^4}{y^2}\right) \div \log_5 w^3$

15. **a.** Since $\log(10x)$ can be written as $\log 10 + \log x$ or $1 + \log x$, the transformation is a dilation horizontally and a vertical translation of one upwards.



b. Since $y = \log_2(8x^2)$ can be written as $y = \log_2 8 + \log_2 x^2$ or $y = 3 + 2\log_2 x$, the transformation is a stretch vertically by a factor of two and a vertical translation of three upwards.



c. Since $y = \log_3(27x^3)$ can be written as $y = \log_3 27 + \log_3 x^3 = 3 + 3\log_3 x$, the transformation is a vertical stretch of three times the original and an upwards vertical translation of three units.

16. **a.** $\log_3\left(27 \bullet \sqrt[3]{81}\right) + \log_5\left(125 \bullet \sqrt[4]{5}\right)$

$= \log_3 27 + \log_3 \sqrt[3]{81} + \log_5 125 + \log_5 \sqrt[4]{5}$

$= 3 + \dfrac{1}{3}\log_3 81 + 3 + \dfrac{1}{4}\log_5 5$

$= 3 + \dfrac{1}{3}(4) + 3 + \left(\dfrac{1}{4}\right)(1)$

$= 6 + \dfrac{4}{3} + \dfrac{1}{4}$

$= 7\,\dfrac{7}{12}$

b. $\log_4\left(2 \bullet \sqrt{32}\right) + \log_{27} \sqrt{3}$

$= \log_4 2 + \log_4 \sqrt{32} + \log_{27} \sqrt{3}$

$= \log_4 4^{\frac{1}{2}} + \log_4 32^{\frac{1}{2}} + \log_{27} 3^{\frac{1}{2}}$

$= \dfrac{1}{2} + \log_4 2^{\frac{5}{2}} + \log_{27} (27^{\frac{1}{3}})^{\frac{1}{2}}$

$= \dfrac{1}{2} + \log_4 (4^{\frac{1}{2}})^{\frac{5}{2}} + \log_{27}(27^{\frac{1}{4}})$

$= \dfrac{1}{2} + \dfrac{5}{4} + \dfrac{1}{6}$

$= \dfrac{23}{12}$

17. **a.** Given $y = 3 \log x$.

(i) If x is multiplied by 2,
then $y = 3 \log 2x$
$= 3[\log 2 + \log x]$
$= 3 \log 2 + 3 \log x$.
So, the value of y increases by $3 \log 2$, or about 0.9.

(ii) If x is divided by 2,
then $y = 3 \log \dfrac{x}{2}$
$= 3[\log x - \log 2]$
$= 3 \log x - 3 \log 2$.
So, the value of y decreases by $3 \log 2$, or 0.9.

b. Given $y = 5 \log x$.

(i) If x is replaced by $4x$, then
$y = 5 \log 4x$
$= 5[\log 4 + \log x]$
$= 5 \log 4 + 5 \log x$,
so that y is increased by $5 \log 4$, or about 3.01.

(ii) If x is replaced by $\dfrac{x}{5}$, then
$y = 5(\log \dfrac{x}{5})$
$= 5[\log x - \log 5]$
$= 5 \log x - 5 \log 5$,
so that y is decreased by $5 \log 5$, or about 3.5.

Exercise 7.3

1. **c.** $2 \log_5 x = \log_5 36$
$\log_5 x^2 = \log_5 36$
$\therefore x^2 = 36$
$x = \pm 6$

The logarithm of a negative number is not defined. The root $x = -6$ is inadmissible.
By inspection $x = 6$ is admissible.

$$\therefore x = 6$$

d. $2 \log x = 4 \log 7$

$$\log x^2 = \log 7^4$$
$$\therefore x^2 = 7^4 \text{ or } (7^2)^2$$
$$x = \pm 49$$

But the logarithm of a negative number is not defined. The root $x = -49$ is inadmissible.
If $x = 49$, L.S. $\doteq 1.690$, R.S. $\doteq 1.690$
$\therefore x = 49$

2. **c.** $2^x - 1 = 4$

$$2x = 5$$
$$\log 2^x = \log 5$$
$$x \log 2 = \log 5$$
$$x = \frac{\log 5}{\log 2}$$
$$\doteq 2.32$$

d. $7 = 12 - 4^x$

$$4^x = 5$$
$$\log 4^x = \log 5$$
$$x \log 4 = \log 5$$
$$x = \frac{\log 5}{\log 4}$$
$$\doteq 1.16$$

3. **a.** $\log x = 2 \log 3 + 3 \log 2$

$$\log x = \log 3^2 + \log 2^3$$
$$\log x = \log 3^2 \times 2^3$$
$$\log x = \log 72$$
$$x = 72$$

c. $\log x^2 = 3 \log 4 - 2 \log 2$

$$\log x^2 = \log 4^3 - \log 2^2$$
$$\log x^2 = \log \frac{4^3}{2^2}$$
$$x^2 = 16$$
$$x = \pm 4$$

Both answers verify, so there are two roots, ± 4.

d. $\log\sqrt{x} = \log 1 - 2 \log 3$

$$\log \sqrt{x} = \log \frac{1}{3^2}$$
$$\sqrt{x} = \frac{1}{9}$$
$$x = \frac{1}{81}$$

e. $\log x^{\frac{1}{2}} - \log x^{\frac{1}{3}} = \log 2$

$$\log\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right) = \log 2$$
$$\log(x^{\frac{1}{6}}) = \log 2$$
$$x^{\frac{1}{6}} = 2$$
$$x = 2^6$$
$$x = 64$$

f. $\log_4(x + 2) + \log_4(x - 3) = \log_4 9$

$$\log_4(x + 2)(x - 3) = \log_4 9$$
$$(x + 2)(x - 3) = 9$$
$$x^2 - x - 6 = 9$$
$$x^2 - x - 15 = 0$$
$$x = \frac{1 \pm \sqrt{1 - 4(1)(-15)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{61}}{2}$$

4. **a.** $\log_6(x + 1) + \log_6(x + 2) = 1$

$$\log_6(x + 1)(x + 2) = 1$$

In exponential form,

$$(x + 1)(x + 2) = 6^1$$
$$x^2 + 3x + 2 = 6$$
$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x = -4 \text{ or } x = 1.$$

But $x = -4$, then $(x + 1) < 0$ and its logarithm is undefined.
$\therefore x = 1$

b. $\log_7(x + 2) + \log_7(x - 4) = 1$

$$\log_7(x + 2)(x - 4) = 1$$

In exponential form,

$$(x + 2)(x - 4) = 7^1$$
$$x^2 - 2x - 8 = 7$$
$$x^2 - 2x - 15 = 0$$
$$(x - 5)(x + 3) = 0$$
$$x = 5 \text{ or } x = -3.$$

But if $x = -3$, $(x + 2) < 0$ and its logarithm is not defined.
$\therefore x = 5$

c. $\log_2(x+2) = 3 - \log_2 x$

$\log_2(x+2) + \log_2 x = 3$

$\log_2 x(x+2) = 3$

In exponential form,

$x(x+2) = 2^3$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ or $x = 2$.

But $\log_2 x$ is not defined for $x = -4$.

$\therefore x = 2$

d. $\log_4 x + \log_4(x+6) = 2$

$\log_4 x(x+6) = 2$

In exponential form,

$x(x+6) = 4^2$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8$ or $x = 2$.

But $\log_4 x$ is not defined for $x = -8$.

$\therefore x = 2$

e. $\log_5(2x+2) - \log_5(x-1) = \log_5(x+1)$

$$\log_5\left(\frac{2x+2}{x-1}\right) = \log_5(x+1),\ x \neq 1$$

$$\frac{2x+2}{x-1} = x+1$$

$2x + 2 = (x+1)(x-1)$

$2x + 2 = x^2 - 1$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

But $\log_5(x+1)$ is not defined for $x = -1$.

$\therefore x = 3$

5. There are no solutions, since logarithms are only defined for a base greater than one and for a number greater than zero, i.e., $\log_6 x$ is defined for $b > 1$ and $x > 0$.

6. If a car depreciates 15% per year, it is worth 85% of its value each year, and so its value can be written as: $V = V_0(0.85)^t$, where t is the time in years.

For half its value, $0.5V_0 = V_0(0.85)^t$

$$0.5 = 0.85^t$$

$$\log 0.5 = t \log 0.85$$

$$t = \frac{\log 0.5}{\log 0.85}$$

$$\doteq 4.265$$

It will depreciate to half of its value in $4\frac{1}{4}$ years or 4 years, 3 months.

7. The amount of radioactive carbon can be modelled by $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{5760}}$, where t is the age in years. For dating of the animal skeleton:

$$\frac{3}{4}A_0 = A_0\left(\frac{1}{2}\right)^{\frac{t}{5760}}$$

$$0.75 = (0.5)^{\frac{t}{5760}}$$

$$\log 0.75 = \frac{t}{5760}\log 0.5$$

$$t = \left[\frac{\log 0.75}{\log 0.5}\right]5760$$

$$\doteq 2390.6$$

The animal skeleton is approximately 2400 years old.

8. Let the time required before replacement be t years.

The amount of Co^{60} is $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{5.24}}$

$$0.45A_0 = A_0(0.5)^{\frac{t}{5.24}}$$

$$0.45 = (0.5)^{\frac{t}{5.24}}$$

$$\log 0.45 = \frac{t}{5.24}\log 0.5$$

$$t = \left[\frac{\log 0.45}{\log 0.5}\right] \times 5.24$$

$$\doteq 6.04$$

The cobalt should be replaced every six years.

9. The amount of carbon^{14} can be modelled as

$$C = \text{Co}(0.5)^{\frac{t}{5760}}$$

$$4.2 \times 10^{10} = 5.0 \times 10^{10}(0.5)^{\frac{t}{5760}}$$

$$0.84 = 0.5^{\frac{t}{5760}}$$

$$\log 0.84 = \frac{t}{5760}\log 0.5$$

$$t = \left[\frac{\log 0.84}{\log 0.5}\right] \times 5760$$

$$t \doteq 1449$$

The relic is only about 1450 years old, so it cannot be authentic.

10. $\log_2(\log_3 a) = 2$
In exponential form,
$\log_3 a = 2^2$
$\log_3 a = 4.$
In exponential form,
$a = 3^4$
$= 81.$

11. $\log_{2n}(1944) = \log_n(486\sqrt{2})$
Let $\log_n(486\sqrt{2}) = x$.
In exponential form,
$n^x = 486\sqrt{2}. \qquad (1)$
Also, $\log_{2n}(1944) = x$.
In exponential form,
$(2n)^x = 1944. \qquad (2)$
Dividing equation (2) by (1), we have

$$\frac{(2n)^x}{n^x} = \frac{1944}{486\sqrt{2}}$$

$$\left(\frac{2n}{n}\right)^x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$2^x = 2^{\frac{3}{2}}$$

$$x = \frac{3}{2} \text{ or } 1.5.$$

Substituting into equation (1):

$$\begin{aligned} n^{1.5} &= 486\sqrt{2} \\ (n^{1.5})^4 &= (486\sqrt{2})^4 \\ n^6 &= 472\,392^2 \\ &\doteq 2.23 \times 10^{11} \end{aligned}$$

Exercise 7.4

2. If an earthquake has a magnitude of five on the Richter scale, then $\log\left[\frac{I_1}{I_0}\right] = 5$ or $I_1 = 10^5 I_0$.
If the second earthquake has a magnitude of six, then $\log\left[\frac{I_2}{I_0}\right] = 6$ or $I_2 = 10^6 I_0$.
Comparing the two, $\frac{I_2}{I_1} = \frac{10^6 I_0}{10^5 I_0}$
$= 10.$
So, the second quake is 10 times as intense as the first.

3. If the sound is 1 000 000 or 10^6 times as loud as one you can just hear, $\frac{I}{I_0} = 10^6$.
The loudness of the sound is given by
$L = 10\log\left[\frac{I}{I_0}\right]$.
The loudness of the sound:
$= 10\log[10^6]$
$= 10(6)$
$= 60.$
The loudness is 60 decibels.

4. The definition of pH is given by

$$\text{pH} = -\log[H^+]$$

For this liquid, $\text{pH} = -\log[8.7 \times 10^{-6}]$
$= -(\log 8.7 + \log 10^{-6})$
$= -(\log 8.7 - 6)$
$= 6 - \log 8.7$
$\doteq 6 - 0.9395.$
The pH is then $\doteq 5.06$.

5. For the earthquake of magnitude two, $I_2 = 10^2 I_0$.
For the earthquake of magnitude four, $I_4 = 10^4 I_0$.
Comparing the intensities,

$$\frac{I_4}{I_2} = \frac{10^4 I_0}{10^2 I_0}$$

$= 10^2.$
So, the larger earthquake is 100 times as intense as the smaller.

6. For the earthquake measuring 4, $I_4 = 10^4 I_0$.
For the earthquake in China measuring 8.6, $I_{8.6} = 10^{8.6} I_0$.
Comparing them, $\frac{I_{8.6}}{I_4} = \frac{10^{8.6} I_0}{10^4 I_0}$
$= 10^{8.6-4}$
$= 10^{4.6}$
or $\doteq 39\,811.$
The earthquake in China was almost 40 000 times as intense as the lesser one.

7. **a.** For the earthquake in Pakistan, $I_P = 10^{6.8} I_0$.
For the earthquake in California, $I_C = 10^{6.1} I_0$.
Comparing $\frac{I_P}{I_C} = \frac{10^{6.8} I_0}{10^{6.1} I_0} = 10^{6.8-6.1} \doteq 5.01.$
The quake in Pakistan was five times as intense as that in California.

8. For the earthquake in Chile, $I_C = 10^{8.3}I_0$.

For the earthquake in Taiwan, $I_T = 10^{7.6}I_0$.

Comparing the two quakes, $\dfrac{I_T}{I_C} = \dfrac{10^{8.3}I_0}{10^{7.6}I_0}$

$= 10^{0.7}$

$\doteq 5.$

The earthquake in Chile was five times more intense.

9. The loudness of a sound is given by $L = 10\log\left(\dfrac{I}{I_o}\right)$.

For her defective muffler,

$120 = 10\log\left(\dfrac{I_d}{I_0}\right)$

$12 = \log\left(\dfrac{I_d}{I_0}\right)$.

Solving for $\dfrac{I_d}{I_0} = 10^{12}$

$I_d = 10^{12}I_0$.

For the new muffler,

$75 = 10\log\left(\dfrac{I_n}{I_0}\right)$

$7.5 = \log\left[\dfrac{I_n}{I_0}\right]$ or $\log\dfrac{I_n}{I_0} = 10^{7.5}$.

So, $I_n = 10^{7.5}I_0$.

Comparing the sounds,

$\dfrac{I_d}{I_n} = \dfrac{10^{12}I_0}{10^{7.5}I_0}$

$= 10^{12-7.5}$

$\doteq 31\ 623$.

So, the sound with a defective muffler is almost 32 000 times as loud as the sound with a new muffler.

10. The loudness level is given by $L = 10\log\left[\dfrac{I}{I_0}\right]$.

For a baby with colic,

$75 = 10\log\left[\dfrac{I_c}{I_0}\right]$ or $\log\left[\dfrac{I_c}{I_o}\right] = 7.5$.

$\therefore I_c = 10^{7.5}I_0$

For a sleeping baby,

$35 = 10\log\left[\dfrac{I_s}{I_0}\right]$ or $\log\left[\dfrac{I_s}{I_o}\right] = 3.5$.

$\therefore I_s = 10^{3.5}I_0$

Comparing the noise level, $\dfrac{I_c}{I_s} = \dfrac{10^{7.5}I_0}{10^{3.5}I_0}$

$= 10^4$.

The noise level with a baby with colic is 10 000 times as loud as when the baby is asleep.

11. For a space shuttle, $I_s = 10^{18}I_0$.

For a jet engine, $I_j = 10^{14}I_0$.

Comparing, $\dfrac{I_s}{I_j} = \dfrac{10^{18}I_0}{10^{14}I_0}$

$= 10^4$ or 10 000.

A space shuttle launch is 10 000 times as loud as a jet engine.

12. For open windows, $I_1 = 10^{7.9}I_0$.

For closed windows, $I_2 = 10^{6.8}I_0$.

Comparing $\dfrac{I_1}{I_2} = \dfrac{10^{7.9}I_0}{10^{6.8}I_0}$

$= 10^{1.1}$ or 12.6.

Closing the windows reduces the noise by a factor of about 13.

13. The pH level is defined by pH = $-\log[H^+]$.

For milk, $6.50 = -\log[H^+]$ or $\log[H^+] = -6.5$

$[H^+] = 10^{-6.5}$

$= 3.2 \times 10^{-7}$

14. The hydrogen ion concentration of milk of magnesia is 3.2×10^{-7} mol/L.

The pH level is defined by pH = $-\log[H^+]$.

For milk of magnesia, $10.50 = -\log[H^+]$

$\log[H^+] = -10.5$

$[H^+] = 10^{-10.5}$

$\doteq 3.2 \times 10^{-11}$.

Milk of magnesia has an ion concentration of 3.2×10^{-1} mol/L.

Exercise 7.5

1. **b.** $\log_7 124 = \dfrac{\log 124}{\log 7}$

$\doteq 2.477$

c. $\log_6 3.24 = \dfrac{\log 3.24}{\log 6}$

$\doteq 0.656$

2. **a.** To prove $\dfrac{1}{\log_5 a} + \dfrac{1}{\log_3 a} = \dfrac{1}{\log_{15} a}$

$$\begin{aligned}
\text{L.S.} &= \frac{1}{\log_5 a} + \frac{1}{\log_3 a} \\
&= log_a 5 + log_a 3 \\
&= log_a (5 \times 3) \\
&= log_a 15 \\
&= \frac{1}{\log_{15} a} \\
&= \text{R.S.}
\end{aligned}$$

$\therefore \dfrac{1}{\log_5 a} + \dfrac{1}{\log_5 a} = \dfrac{1}{\log_{15} a}$

b. To prove $\dfrac{1}{\log_8 a} - \dfrac{1}{\log_2 a} = \dfrac{1}{\log_4 a}$

$$\begin{aligned}
\text{L.S.} &= \frac{1}{\log_8 a} - \frac{1}{\log_2 a} \\
&= \log_a 8 - \log_a 2 \\
&= \log_a \left(\frac{8}{2}\right) \\
&= \log_a 4 \\
&= \frac{1}{\log_4 a} \\
&= \text{R.S.}
\end{aligned}$$

$\therefore \dfrac{1}{\log_8 a} - \dfrac{1}{\log_2 a} = \dfrac{1}{\log_4 a}$

c. To prove $\dfrac{2}{\log_6 a} = \dfrac{1}{\log_{36} a}$

$$\begin{aligned}
\text{L.S.} &= \frac{2}{\log_6 a} \\
&= 2(\log_a 6) \\
&= \log_a 6^2 \\
&= \log_a 36 \\
&= \frac{1}{\log_{36} a} \\
&= \text{R.S.}
\end{aligned}$$

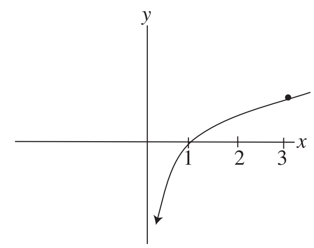
$\therefore \dfrac{2}{\log_6 a} = \dfrac{1}{\log_{36} a}$

d. To prove $\dfrac{2}{\log_8 a} - \dfrac{4}{\log_2 a} = \dfrac{1}{\log_4 a}$

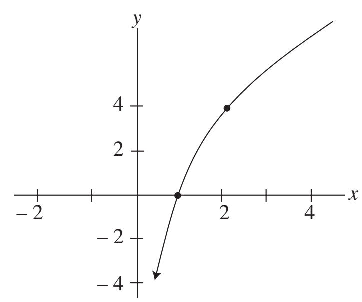
$$\begin{aligned}
\text{L.S.} &= \frac{2}{\log_8 a} - \frac{4}{\log_2 a} \\
&= 2\log_a 8 - 4\log_a 2 \\
&= \log_a 8^2 - \log_a 2^4 \\
&= \log_a \left(\frac{8^2}{2^4}\right) \\
&= \log_a 4 \\
&= \frac{1}{\log_4 a} \\
&= \text{R.S.}
\end{aligned}$$

$\therefore \dfrac{2}{\log_8 a} - \dfrac{4}{\log_2 a} = \dfrac{1}{\log_4 a}$

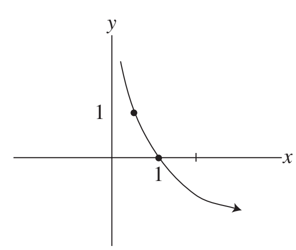
3. **a.** $y = \log_3 x$



b. $y = \log_{0.5} x$

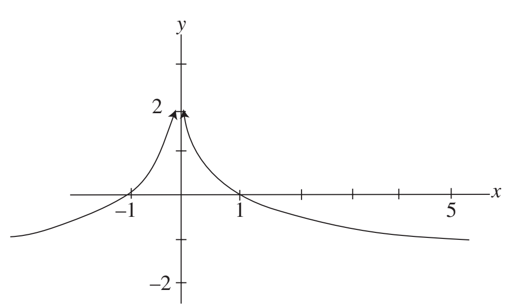


c. $y = 4\log_2 x$



d. $y = \log_{0.2} x^2$

Since x^2 is always positive, we can include negative values for x as well.



Solution 1

Given $a > 1$, $b > 1$,
show $(\log_a b)(\log_b a) = 1$.
Proof:

$$\log_a b = \frac{1}{\log_b a}$$

$$\therefore \text{ L.S.} = (\log_a b)(\log_b a)$$
$$= \frac{1}{\log_b a} \times \log_b a$$
$$= 1$$
$$= \text{R.S.}$$

Solution 2

$$(\log_a b)(\log_b a)$$
$$= (\log_a b) \times \frac{1}{(\log_a b)}$$
$$= 1$$
$$= \text{R.S.}$$

6. Noting that the L.S. has $(a + b)$, we find an expression for it in terms of $a^2 + b^2$.

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$\therefore\ a^2 + b^2 = (a + b)^2 - 2ab$$

But, we are given that $a^2 + b^2 = 23ab$.

$$\therefore\ (a + b)^2 - 2ab = 23ab$$
$$(a + b)^2 = 25ab \quad \text{or} \quad \frac{(a + b)^2}{25} = ab$$

That is $\left[\frac{(a + b)}{5}\right]^2 = ab$.

Taking logarithms of both sides, we have

$$\log\left[\frac{(a + b)}{5}\right]^2 = \log ab$$
$$2\log\left(\frac{a + b}{5}\right) = \log a + \log b$$
$$\therefore\ \log\left(\frac{a + b}{5}\right) = \frac{1}{2}(\log a + \log b).$$

7. $\log_a \frac{1}{x} = \log_{\frac{1}{a}} x$

Let $\log_a \frac{1}{x} = b$.

Then, in exponential form

$$a^b = \frac{1}{x}$$
$$a^b = x^{-1} \quad \text{or} \quad x = a^{-b}.$$

Taking logarithms of both sides, we have

$$\log_{\frac{1}{a}} x = \log_{\frac{1}{a}} a^{-b}$$
$$= -b \log_{\frac{1}{a}} a$$
$$= -b \log_{\frac{1}{a}}\left(\frac{1}{a}\right)^{-1}$$
$$= -b(-1)$$
$$= b$$
$$= \log_a\left(\frac{1}{x}\right).$$

8. *Solution 1*

$$\log_a b = p^3$$

In exponential form, $a^{p^3} = b$ (1)

$$\log_b a = \frac{4}{p^2}$$

In exponential form $b^{\frac{4}{p^2}} = a$ (2)

Substituting for b from equation (1):

$$\left(\frac{p^3}{a}\right)^{\frac{4}{p^2}} = a$$
$$a^{4p} = a^1$$
$$\therefore\ 4p = 1$$
$$p = \frac{1}{4}$$

Solution 2

$$\log_a b = p^3$$
$$\therefore\ \log_b a = \frac{1}{p^3}$$

But, $\log_b a = \frac{4}{p^2}$:

$$\therefore\ \frac{4}{p^2} = \frac{1}{p^3}$$
$$4p = 1$$
$$p = \frac{1}{4}.$$

Solution 3

Since $\log_a b = p^3$,

$$\frac{1}{\log_a b} = \frac{1}{p^3}$$

$$\log_b a = \frac{1}{p^3}$$

$$\frac{4}{p^2} = \frac{1}{p^3}$$

$$4p = 1$$

$$p = \frac{1}{4}.$$

9. Noting that $a^3 - b^3$ is given, but $a - b$ is required, we find $(a - b)$ in terms of $a^3 - b^3$.

Since $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$,

$$(a - b)^3 = (a^3 - b^3) - 3a^2b + 3ab^2.$$

But, it is given that $a^3 - b^3 = 3a^2b + 4ab^2$,

$$\therefore (a - b)^3 = (3a^2b + 5ab^2) - 3a^2b + 3ab^2$$

$$(a - b)^3 = 8ab^2$$

$$\frac{(a - b)^3}{8} = ab^2$$

$$\left(\frac{a - b}{2}\right)^3 = ab^2.$$

Taking the logarithms of both sides,

$$\log\left(\frac{a - b}{2}\right)^3 = \log(ab^2)$$

$$3\log\left(\frac{a - b}{2}\right) = \log a + 2\log b$$

$$\log\left(\frac{a - b}{2}\right) = \frac{1}{3}(\log a + 2\log b).$$

Review Exercise

2. c. $\log_5 \sqrt[3]{25} - \log_3 \sqrt{27}$

$= \log_5 25^{\frac{1}{3}} - \log_3 27^{\frac{1}{3}}$

$= \log_5 (5^2)^{\frac{1}{3}} - \log_3 (3^3)^{\frac{1}{3}}$

$= \log_5 5^{\frac{2}{3}} - \log_3 3$

$= \frac{2}{3} - 1 \quad \text{or} \quad -\frac{1}{3}$

d. $7^{\log_7 5}$

Let $\log_7 5 = x$.

$$7^x = 5$$

$$\therefore 7^{\log_7 5} = 7^{\log_7 7x}$$

$$= 7^x$$

$$= 5$$

3. b. $\log(x + 3) + \log x = 1$

$\log x(x + 3) = 1$

In exponential form:

$$x(x + 3) = 10^1$$

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$x + 5 = 0 \quad \text{or} \quad x - 2 = 0$

$x = -5 \quad \text{or} \quad x = 2$

But $\log x$ is not defined for $x = -5$,

$\therefore x = 2.$

c. $\log_5(x + 2) - \log_5(x - 1) = 2\log_5 3$

$$\log_5\left[\frac{x + 2}{x - 1}\right] = \log_5 3^2,\ x \neq 1$$

$$\frac{x + 2}{x - 1} = 9$$

$$x + 2 = 9x - 9$$

$$-8x = -11$$

$$x = \frac{11}{8}$$

d. $\dfrac{\log(35 - x^3)}{\log(5 - x)} = 3$

$$\log(35 - x^3) = 3\log(5 - x)$$

$$\log(35 - x^3) = \log(5 - x)^3$$

$$35 - x^3 = (5 - x)^3$$

$$35 - x^3 = 125 - 3(5)^2x + 3(5)x^2 - x^3$$

$$35 - x^3 = 125 - 75x + 15x^2 - x^3$$

$$15x^2 - 75x + 90 = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$x - 3 = 0 \quad \text{or} \quad x - 2 = 0$

$x = 3 \quad \text{or} \quad x = 2$

4. For an earthquake of 7.2, the intensity is $I_J = 10^{7.2}I_0$.

For an earthquake of 6.9, the intensity is $I_A = 10^{6.9}I_0$.

$$\text{Comparing, } \frac{I_J}{I_A} = \frac{10^{7.2}I_0}{10^{6.9}I_0}$$
$$= 10^{7.2-6.9}$$
$$= 10^{0.3}$$
$$\doteq 1.99.$$

The earthquake in Kobe was twice as intense as that in Armenia.

5. Loudness of sound is given by $L = 10\log\left(\frac{I}{I_0}\right)$, where L is in decibels and I is the intensity.

$$\text{Morning noise is } 50 = 10\log\left(\frac{I_M}{I_0}\right)$$
$$5 = \log\left(\frac{I_m}{I_0}\right) \quad \text{or} \quad \frac{I_m}{I_0} = 10^5$$
$$I_m = 10^5 I_0.$$

$$\text{Similarly for noon noise: } 100 = 10\log\left(\frac{I_N}{I_0}\right)$$
$$10 = \log\left(\frac{I_n}{I_0}\right)$$
$$I_n = 10^{10}I_0.$$

$$\text{Comparing, } \frac{I_n}{I_m} = \frac{10^{10}I_0}{10^5 I_0}$$
$$= 10^5.$$

The noise at noon in the cafeteria is 10^5 or 100 000 times as loud as in the morning.

6. pH is defined as pH = $-\log[H^+]$.

For this liquid, $5.62 = -\log[H^+]$:

$$\log[H^+] = -5.62$$
$$[H^+] = 10^{-5.62}$$
$$\doteq 2.3988 \times 10^{-6}.$$

The hydrogen ion concentration is approximately 2.4×10^{-6} moles/L.

8. **a.** $\log_{19}264$

$$= \frac{\log 264}{\log 19}$$
$$\doteq 1.894$$

b. $\log_5 34.62$

$$= \frac{\log 34.62}{\log 5}$$
$$\doteq 2.202$$

9. $$\frac{2}{\log_9 a} - \frac{1}{\log_3 a} = \frac{3}{\log_3 a}$$

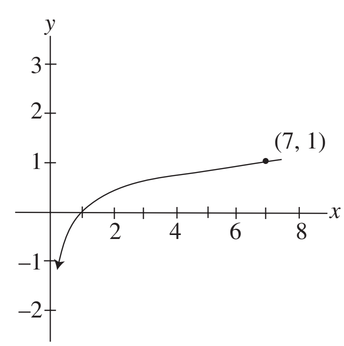
$$\text{L.S.} = \frac{2}{\log_9 a} - \frac{1}{\log_3 a}$$
$$= 2\log_a 9 - \log_a 3$$
$$= \log_a 9^2 - \log_a 3$$
$$= \log_a \frac{81}{3}$$
$$= \log_a 27$$

$$\text{R.S.} = \frac{3}{\log_3 a}$$
$$= 3\log_a 3$$
$$= \log_a 3^3$$
$$= \log_a 27$$

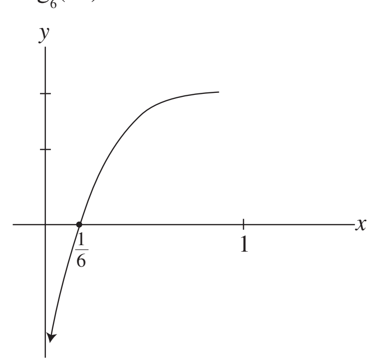
L.S. = R.S.

$$\therefore \frac{2}{\log_a a} - \frac{1}{\log_3 a} = \frac{3}{\log_3 a}$$

10. **a.** $y = \log_7 x$



b. $y = 2\log_6(6x)$



Chapter 7 Test

1. **a.** $\log_3 27 = 3$
$(3^3 = 27)$

b. $\log_5 125 = 3$
$(5^3 = 125)$

c. $\log_2 \frac{1}{16} = -4$
$\left(2^{-4} = \frac{1}{16}\right)$

d. $\log_5 \sqrt[4]{25}$
$= \log_5 25^{\frac{1}{4}}$
$= \frac{1}{4} \log_5 25$
$= \frac{1}{4} \times 2$
$= \frac{1}{2}$

e. $\log_2 8 + \log_3 9$
$= 3 + 2$
$= 5$

f. $\log_3 9^{\frac{1}{3}}$
$= \frac{1}{3}\log_3 9$
$= \frac{1}{3} \times 2$
$= \frac{2}{3}$

2. **a.** $\log_2 \frac{8}{5} + \log_2 10$
$= \log_2 \frac{8}{5} \times 10$
$= \log_2 16$
$= 4\ (2^4 = 16)$

b. $\log_6 108 - \log_6 3$
$= \log_6 \frac{108}{3}$
$= \log_6 36$
$= 2\ (6^2 = 36)$

3. Vertical stretch by a factor of two, translated two units up.

4. **a.** $2 \log x = 3 \log 4$
$\log x = \frac{3}{2} \log 4$
$\log x = \log 4^{\frac{3}{2}}$
$x = 4^{\frac{3}{2}}$
$x = 8$

b. $\log x + \log 3 = \log 12$
$\log 3x = \log 12$
$3x = 12$
$x = 4$

c. $\log_2(x + 2) + \log_2 x = 3$
$\log_2 x(x + 2) = 3$
$x(x + 2) = 2^3$
$x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$x = -4$ or $x = 2$
But $x > 0$, therefore $x = -4$ is inadmissible.
Verify $x = 2$.
L.S. $= \log_2 4 + \log_2 2$
$= \log_2 8$
$= 3$
$=$ R.S.
Therefore, $x = 2$.

d. $\log_2(x - 2) + \log_2(x + 1) = 2$
$\log_2(x - 2)(x + 1) = 2$
$x^2 - x - 2 = 4$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3$ or $x = -2$
If $x = 3$,
L.S. $= \log_2 1 + \log_2 4$
$= \log_2 4$
$= 2$
$=$ R.S.
If $x = -2$,
L.S. $= \log_2 0 + \log_2(-1)$, which is not possible.
Therefore, $x = -2$ is inadmissible, and the answer is $x = 3$.

5. $\log_3(-9) = x$

or $3^x = -9$

There is no real value for x such that a power of 3 is a negative number.

6. The formula for half-life is given by $A(t) - A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$.

$A_0 = 20$

$A(t) = 15$

$t = 7$

$$15 = 20\left(\frac{1}{2}\right)^{\frac{7}{h}}$$

$$\frac{15}{20} = \left(\frac{1}{2}\right)^{\frac{7}{h}}$$

Take the logarithm of both sides by

$15 - \log 20 = \dfrac{7}{h}\log 0.5$:

$h(\log 15 - \log 20) = 7 \log 0.5$

$$h = \frac{7 \log 0.5}{\log 15 - \log 20}$$

$= 16.87.$

The half-life is 16.87 h.

7. For the earthquake in Tokyo, $I_T = 10^{8.3}I_0$.

For the earthquake in Guatemala, $I_G = 10^{7.5}I_0$.

Comparing the two earthquakes, $\dfrac{I_T}{I_G} = \dfrac{10^{8.3}I_0}{10^{7.5}I_0}$

$= 10^{0.8}$

$\doteq 6.3.$

The earthquake in Tokyo was six times more intense than the earthquake in Guatemala.

8. The loudness of sound is given by $L = 10\log\left(\dfrac{I}{I_0}\right)$.

For the subway platform, $60 = 10\log\left(\dfrac{I_s}{I_0}\right)$

$\dfrac{I_s}{I_0} = 10^6$

$I_s = 10^6 I_0$

For the subway train, $90 = 10\log\left(\dfrac{I_r}{I_0}\right)$

$\log\left(\dfrac{I_T}{I_0}\right) = 9$

$\dfrac{I_T}{I_0} = 10^9$

Then, $I_T = 10^9 I_0$.

Comparing the sounds, $\dfrac{I_T}{I_S} = \dfrac{10^9 I_0}{10^6 I_0}$

$= 10^3$

$= 1000.$

The noise level is 1000 times more intense when the train arrives.

9. The pH level is defined by $pH = -\log[H^+]$.

For the liquid, $8.31 = -\log[H^+]$

$\log[H^+] = -8.31$

$H^+ = 10^{-8.31}$

$= 4.90 \times 10^{-9}.$

The hydrogen ion concentration is 4.9×10^{-9} moles/L.

10. Prove $\dfrac{3}{\log_2 a} = \dfrac{1}{\log_8 a}$.

L.S. $= \dfrac{3}{\log_2 a}$

$= 3\log_a 2$

R.S. $= \dfrac{1}{\log_8 a}$

$= \log_a 8$

$= \log_a 2^3$

$= 3\log_a 2$

Therefore, $\dfrac{3}{\log_2 a} = \dfrac{1}{\log_8 a}$.

11. $\log_a b = \dfrac{1}{x}$ and $\log_b \sqrt{a} = 3x^2$, $x = \dfrac{1}{6}$

$\log_a b = \dfrac{1}{x}$

$a^{\frac{1}{x}} = b$

$a = b^x$

$\log_b \sqrt{a} = 3x^2$

$b^{3x^2} = a^{\frac{1}{2}}$

$a = b^{6x^2}$

Therefore, $b^{6x^2} = b^x$ and $6x^2 = x$.

But $x \neq 0$, therefore, $6x = 1$,

$x = \dfrac{1}{6}$, as required.